

Homework 0

CS304: Automata and Formal Languages

July 29, 2025

Question 1. Set Notation. Let $A = \{a, b, \{c, d\}, e\}$ and $B = \{a, b, c, d\}$. Which of the following statements are true?

- $c \in A$
- $\{c, d\} \in A$
- $\{\{c, d\}\} \in A$
- $c \in B$
- $|A| = 5$
- $|A| = |B|$
- $A \setminus B = \{e\}$

Question 2. Set Operations. Let the universal set be $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Consider sets $P = \{x \mid x \text{ is a prime number}\}$ and $E = \{x \mid x \text{ is an even number}\}$. List the elements of the following sets:

- $P \cap E$
- $P \cup E$
- $P \setminus E$
- $\overline{E}$

Question 3. Boolean Logic. Construct a truth table to verify the truth values for the logical expression $(p \implies q) \wedge (\neg(p \vee \neg q))$.

Question 4. Combinatorics. Suppose we have Rs. 10,000 to invest in 4 mutual funds, where each mutual fund requires investments in multiples of Rs. 1,000. In how many ways can we do this?

Question 5. Proof. Prove using induction that for all $n \geq 0$, the number $4^{2n+1} + 3^{n+2}$ is divisible by 13.