Homework 1

CS304: Automata and Formal Languages

Due: Mon Aug 25, 2025

Submission Guidelines: Your submission must be a **single PDF file**. You are strongly encouraged to typeset your solutions using LaTeX, as it is the standard in computer science. (Overleaf is a great starting point for beginners.) However, solutions written in any other word processor (like MS Word or Google Docs) are acceptable, as long as you export the final document to PDF. For DFA diagrams you have two options:

- (**preferred**) Use a drawing tool (e.g., draw.io, TikZ, or any other vector-graphics editor) and embed the vector-image directly into your document.
- Draw the diagram neatly on paper and take a clear, well-lit photograph. Crop the resulting image and embed it legibly within your document.

Question 1 (40 points)**.** *Provide the diagram of a DFA (following the conventions used in class) accepting the following languages.*

- $\Sigma = \{a, b, c, \ldots, z\}$ *and* $L = \{w \mid$ *'iiit' is a substring of* $w\}$
- $\Sigma = \{0, 1\}$ *and* $L = \{w \mid w$ *is a binary representation of a number divisible by* $3\}$[1]

Question 2 (40 points)**.** *Consider a DFA* $A = \left\{Q, \Sigma, \delta, q_0, \left\{q_f\right\}\right\}$ *such that for all* $a \in \Sigma$, $\delta(q_0, a) = \delta(q_f, a)$*. Prove using induction that if a string* w *over* Σ *is accepted by* A*, then for all* $k > 0$*, the string* w^k *is also accepted by* A*. Clearly write the base case, the induction hypothesis, and the induction step.*

Question 3 (20 points)**.** *Friends in a party.*

- ***The Small Party.*** *There are* 4 *people at a party. Each person counts how many other people at the party they are friends with. Is it possible for each of the* 4 *people to have a different number of friends at the party? Explain why or why not with a clear argument.*
- ***The Variable Party.*** *There are* N *people at a party. (*$N > 1$*, duh.) Each person counts how many other people at the party they are friends with. Is it possible for each of the* N *people to have a different number of friends at the party? If possible, provide an example. If not, provide a proof.*
- *You have just used a powerful logical rule to solve the two problems above. Describe the general principle you discovered in your own words. When does this rule apply? Create a different, simple example of a problem that could be solved with the same principle. (For instance, think about birthdays in a classroom, or pigeons in pigeonholes.)*

[1] Assume the standard big-endian binary representation with possible leading zeros, i.e. $w = 100$ represents the number 4 and $w = 000110$ represents the number 6.