CS304 - Quiz 1

Aug 5, 2025

Name: **Roll No.:**

Question 1. *Prove using induction that for all integers $n \geq 1$, we have $(n+1)^{n-1} \leq n^n$.*

Proof. Since $n \geq 1$, the above statement is equivalent to proving that for all $n \geq 1$,

$$\frac{(n+1)^{n-1}}{n^{n-1}} \leq \frac{n^n}{n^{n-1}} \qquad \text{(divide both sides by } n^{n-1}\text{)}$$

$$\text{or, } \left(1+\frac{1}{n}\right)^{n-1} \leq n$$

We proceed by induction on n.

Base Case: For $n = ...$,

Induction Hypothesis: For some $k \geq 1$, suppose $\left(1+\frac{1}{k}\right)^{k-1} \leq k$.

Inductive Step: We now need to show that,

$$\left(1+\frac{1}{...}\right)^{...} \leq ...$$

Conclusion: By the principle of induction, for all integers $n \geq 1$, we have shown that $(n+1)^{n-1} \leq n^n$ holds. $\square$

Rough Space.